

Infinitesimally Nonlocal Lorentz Violation

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Abstract

We introduce a new Lorentz-violating modification to a scalar quantum field theory. This interaction, while super-renormalizable by power counting, is fundamentally different from the interactions previously considered within the Lorentz-violating standard model extension. The Lagrange density is nonlocal, because of the presence of a Hilbert transform term; however, this nonlocality is also very weak. The theory has reasonable stability and causality properties and, although the Lorentz-violating interaction possesses a single vector index, the theory is nonetheless CPT even. As an application, we analyze the possible effects of this new form of Lorentz violation on neutral meson oscillations. We find that under certain circumstances, the interaction may lead to quite peculiar sidereal modulations in the oscillation frequency.

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Recent work has stimulated a great deal of interest in the possibility of there existing small Lorentz- and CPT-violating corrections to the standard model. Such violations of fundamental symmetries may arise as part of the low-energy behavior of the novel physics of the Planck scale. The general local Lorentz-violating standard model extension (SME) has been developed [1, 2, 3], and the stability [4] and renormalizability [5] of this extension have been studied. The minimal SME includes superficially renormalizable operators that are invariant under the standard model gauge group.

The SME provides a good framework within which to analyze the results of experiments testing Lorentz violation. To date, such experimental tests have included studies of matter-antimatter asymmetries for trapped charged particles [6, 7, 8, 9] and bound state systems [10, 11], determinations of muon properties [12, 13], analyses of the behavior of spin-polarized matter [14, 15], frequency standard comparisons [16, 17, 18], measurements of neutral meson oscillations [19, 20, 21, 22], polarization measurements on the light from distant galaxies [23, 24, 25], and others.

However, there are other operators, beyond those considered in the SME, that might prove important in the ultimate low-energy effective field theory describing Lorentz violation. By considering theories with Lorentz violation, we are relaxing the usual conditions that one places on the Lagrangian of a quantum field theory. It is therefore natural to consider, in conjunction with Lorentz violation, slight relaxations of other standard conditions, such as locality. In fact, nonlocality is already present in a renormalizable field theory that is regulated with a momentum cutoff, because the action of such a regulator is itself nonlocal. Nonlocality has also previously been suggested as a potentially desirable property of high-energy Lorentz-violating theories [4].

We shall consider a particular nonlocal operator that may be added to a Lorentz invariant Lagrangian. The nonlocality in this operator possesses an inherently Lorentz-violating structure. However, that nonlocality is also very weak—“infinitesimal,” in a particular sense that we shall describe. It is also significant that the theory we shall consider, although it possesses a nonlocal Lagrangian, does not appear to violate any causality conditions.

Our infinitesimally nonlocal variety of Lorentz violation involves the appearance of the Hilbert transform, which, in one dimension, takes the form

$$Hu(x) = \mathcal{P} \frac{1}{\pi} \int d\xi \frac{u(\xi)}{\xi - x}, \quad (1)$$

where \mathcal{P} denotes the principal value of the integral. In the Fourier transform domain, the Hilbert transform generates a $\frac{\pi}{2}$ phase shift (i.e. $H \sin kx = \cos kx$ and $H \cos kx = -\sin kx$ if $k > 0$). When coupled with a derivative, this phase shift can give rise to very interesting dispersion relations, according to

$$\partial_x (H e^{ikx}) = H(\partial_x e^{ikx}) = -|k| e^{ikx}. \quad (2)$$

Dispersion relations based on (2) appear in other areas of physics, most notably in the Benjamin-Ono equation in fluid mechanics [26],

$$u_t + c_1 u_x + c_2 u u_x + c_3 H u_{xx} = 0. \quad (3)$$

The Hilbert transform also arises in signal processing and in the study of complex dispersion relations.

In the context of quantum field theory, we may introduce a similar Lorentz-violating modification of the dispersion relation. The Lagrange density appropriate to a scalar field theory incorporating such a modification is

$$\mathcal{L}_H = \frac{1}{2}(\partial^\mu \phi)(\partial_\mu \phi) - \alpha a_H^\mu [\phi (H_{\hat{a}_H} \partial_\mu \phi)] - \frac{m^2}{2} \phi^2. \quad (4)$$

To avoid any complexities associated with the appearance of nonstandard time derivatives, we shall take a_H^μ to be purely spacelike in the frame in which the theory is quantized. Otherwise, we would have a Lagrangian that would be nonlocal in time, and this is clearly inadmissible in canonical quantum theory. (Moreover, if a timelike component for a_H^μ were allowed, we would need to define the theory in Wick-rotated Euclidean space.) $H_{\hat{a}_H}$ represents the Hilbert transform taken along the direction of the unit vector $\hat{a}_H = \vec{a}_H/|\vec{a}_H|$,

$$H_{\hat{a}_H} \phi(t, \vec{x}) = \mathcal{P} \frac{1}{\pi} \int d\xi \frac{\phi[t, \vec{x} - (\hat{a}_H \cdot \vec{x}) \hat{a}_H + \xi]}{\xi - \hat{a}_H \cdot \vec{x}}. \quad (5)$$

α is either ± 1 and determines the sign of the novel term. The interactions we are considering are by no means the most general that may be represented by the Hilbert transform. However, (4), which contains “collinear” operators $\vec{a}_H \cdot \vec{\partial}$ and $H_{\hat{a}_H}$, represents the most natural of the possible Lagrangians that could be considered.

It is also possible to envision a reasonable scenario in which an interaction such as $a_H^\mu [\phi (H_{\hat{a}_H} \partial_\mu \phi)]$ could arise in an effective field theory. The Benjamin-Ono equation (3) describes the waves and solitons that occur within a certain regime of fluid mechanics. The underlying equations of motion (i.e. the Navier-Stokes equations) do not involve Hilbert transforms, but the effective theory does. We envision something analogous in the particle physics situation. For example, if the elementary particles we observe are actually the solitary waves of some underlying theory, then, as in the Benjamin-Ono case, the dynamics of these solitary waves could be determined by interactions involving Hilbert transforms. In the fundamental theory, the absolute value appearing in the dispersion relation could be replaced by a smooth function, differing from the absolute value only in a region of momentum space whose size is suppressed by some power of a large mass scale. The coefficient a_H^μ could arise in the same way as any other Lorentz-violating parameter, perhaps as the vacuum expectation value of a vector-valued field.

The theory with a_H^μ is closely related to a simpler Lorentz-violating theory, defined by

$$\mathcal{L}_a = (\partial^\mu \Phi^*) (\partial_\mu \Phi) + i a^\mu [\Phi^* (\partial_\mu \Phi) - (\partial_\mu \Phi^*) \Phi] - m^2 \Phi^* \Phi, \quad (6)$$

where Φ is a complex scalar field. For instance, the a^μ and a_H^μ theories have very similar stability conditions, as we shall see. a^μ and a_H^μ both have dimension (mass)¹, and so both are superficially super-renormalizable. However, the a^μ theory is significantly simpler in a number of ways.

Equation (2) tells us that the nonlocality due to the presence of the Hilbert transform is very weak. For a wave packet that is well localized in momentum space, the nonlocality of the dispersion relation will not be evident, since only the Fourier modes with a single sign of $k_{\hat{a}_H} \equiv \vec{k} \cdot \hat{a}_H$ will contribute substantially. Seen another way, the nonlocality only manifests itself in the neighborhood of $k_{\hat{a}_H} = 0$. This breakdown of nonlocality is analogous to the breakdown of associativity that occurs in the presence of a magnetic monopole, when the Jacobi identity fails at a single point in space [27, 28, 29]. Since the breakdown of the theory's expected properties occurs only on a set of measure zero in each case, the theory can remain physically reasonable.

The discrete symmetries of the perturbation $[\phi(H_{\hat{a}_H}\partial_\mu\phi)]$ may be easily worked out. Since we are only considering a purely spacelike a_H^μ , the derivative operator is odd under P and even under C and T. The Hilbert transform has the same symmetries. The sign change of $H_{\hat{a}_H}$ under parity comes from the denominator of (5). The behavior under time reversal can be determined from the facts that $H_{\hat{a}_H}$ does not interact with the time coordinate and that the Hilbert transform is entirely real, so there are no sign changes arising from the antiunitarity of T. Charge conjugation is of course trivial in a real scalar field theory. This means that the coefficient a_H^μ describes a Lorentz-violating parameter with an odd number of Lorentz indices, which is nonetheless CPT-even. There are no local Lorentz-violating interactions with this property.

Because it is even under CPT, a_H^μ cannot contribute radiatively to any of the Lorentz-violating coefficients in the SME at leading order. Conversely, none of the SME parameters will contribute to the renormalization of a_H^μ ; any radiative corrections must be proportional to a_H^μ itself. In fact, if we endow the scalar field in (4) with a ϕ^4 interaction, there will be no one-loop radiative corrections to a_H^μ at all, just as there is no field strength renormalization in the usual ϕ^4 theory at leading order. The only one-loop self-energy diagram is a tadpole, with no dependence on the external momentum. This diagram contributes a mass renormalization and nothing more. So the effective value of a_H^μ is not sensitive to any lowest-order radiative effects.

The stability properties of this theory can be easily verified in the quantization frame. For $\alpha = 1$, the contribution to the energy from the Hilbert transform term is always positive, so stability is assured. If α is negative, then the theory is stable provided that $|\vec{a}_H| \leq m$. This is essentially the same stability condition that arises in the presence of the local $(a \cdot \partial)$ interaction from \mathcal{L}_a . In either case, the condition ensures that the potentially negative Lorentz-violating contributions cannot dominate the energy. We shall naturally consider only stable situations in this paper, and, in any case, we expect that $\frac{|\vec{a}_H|}{m}$ should be very small in any physically relevant scenario.

The question of causality is trickier. However, we can demonstrate one important

result in this area. We consider the group velocity \vec{v}_g for the particles under consideration. For a wave packet well localized around three-momentum \vec{k}_0 , $\vec{v}_g = \vec{\nabla}_{\vec{k}} E(\vec{k})|_{\vec{k}=\vec{k}_0}$. Away from $k_{\hat{a}_H} = 0$, the absolute value in the dispersion relation

$$E(\vec{k}) = \sqrt{\vec{k}^2 + 2\alpha|\vec{a}_H||k_{\hat{a}_H}| + m^2} \quad (7)$$

is not evident, and the group velocities are equivalent to those in the theory defined by \mathcal{L}_a . The \mathcal{L}_a theory possesses full microcausality; this is verified explicitly in [4] for a fermionic version of the a^μ theory, related to the bosonic one by supersymmetry [30]. In the vicinity of $k_{\hat{a}_H} = 0$, the group velocity in the a_H^μ theory ceases to be a well-defined concept; however, while $\partial E/\partial k_{\hat{a}_H}$ is discontinuous, its magnitude is bounded by unity (provided, of course, that $|\vec{a}_H| \leq m$).

The discontinuous behavior of the group velocity in this system has some interesting effects. For a wave packet moving in the plane perpendicular to \vec{a}_H , the momentum spread in the \hat{a}_H -direction is centered around $k_{\hat{a}_H} = 0$. The components of the wave packet with $\text{sgn}(k_{\hat{a}_H}) > 0$ will have a small velocity in the \hat{a}_H -direction of $\frac{\alpha}{m}\vec{a}_H$. The Fourier components with negative $k_{\hat{a}_H}$ will have velocity $-\frac{\alpha}{m}\vec{a}_H$. So no matter how well-localized the wave packet, there will be a discontinuity of $\Delta\vec{v} = \frac{2}{m}\vec{a}_H$ in the velocity, and this will affect wave packet spreading. Over time, the wave packet will bifurcate, with the two halves moving apart with velocity $\Delta\vec{v}$.

Since the form of Lorentz violation we are considering is invariant under C, P, and T, it can be difficult to find an experimental signature for the effects of a_H^μ . However, like an a^μ interaction, these new physics could cause changes in the structure of neutral meson oscillations. CPT violations in the K^0 system have been strongly constrained [31, 32], but CPT-even Lorentz violations are still possible. Moreover, similar effects are also possible for the other neutral mesons— D^0 , B_d^0 , and B_s^0 [33].

In what follows, we shall neglect any CP-violating interactions that are present in the kaon system, and we shall also neglect decay processes, although either of these could be included without much difficulty. To study the effects of a_H^μ on kaon oscillations, we must generalize the Lagrange density (4) to one involving a complex scalar field K . The simplest generalization is

$$\mathcal{L}_{K,0} = (\partial^\mu K^*)(\partial_\mu K) - 2\alpha a_H^\mu [K^*(H_{\hat{a}_H}\partial_\mu K)] - m^2 K^* K. \quad (8)$$

Since the Hilbert transform is purely real, the breakdown of the Lorentz-violating term in terms of $K_1 = \frac{1}{\sqrt{2}}(\phi + \phi^*)$ and $K_2 = \frac{1}{i\sqrt{2}}(\phi - \phi^*)$ is

$$2\alpha a_H^\mu [K^*(H_{\hat{a}_H}\partial_\mu K)] = \alpha a_H^\mu [K_1(H_{\hat{a}_H}\partial_\mu K_1) + K_2(H_{\hat{a}_H}\partial_\mu K_2)]. \quad (9)$$

However, $\mathcal{L}_{K,0}$ does not cause any K^0 - \bar{K}^0 oscillations. The normal oscillations are generated by a mass difference between K_1 and K_2 , and we must include this effect. There is also potentially a small difference between the two values of a_H^μ in (9) corresponding to

the two fields K_1 and K_2 . Like the mass differences, the small differences in a_H^μ could be due to differences in the interaction of the K_1 and K_2 particles with other virtual species. The ultimate Lagrange density we shall consider is therefore

$$\begin{aligned}\mathcal{L}_K = & \frac{1}{2}(\partial^\mu K_1)(\partial_\mu K_1) + \frac{1}{2}(\partial^\mu K_2)(\partial_\mu K_2) - \frac{m_1^2}{2}K_1^2 - \frac{m_2^2}{2}K_2^2 \\ & - \alpha a_{H1}^\mu [K_1 (H_{\hat{a}_{H1}} \partial_\mu K_1)] - \alpha a_{H2}^\mu [K_2 (H_{\hat{a}_{H2}} \partial_\mu K_2)].\end{aligned}\quad (10)$$

The usual oscillations are generated by the beat frequency $\Delta m \equiv m_1 - m_2$. The leading Lorentz-violating modifications may be controlled either by $\frac{\Delta m}{m}|\vec{a}_H|$ or by $|\Delta \vec{a}_H| \equiv |\vec{a}_{H1} - \vec{a}_{H2}|$, depending on whether $\frac{\Delta m}{m}$ or $\frac{|\Delta \vec{a}_H|}{|\vec{a}_H|}$ is larger. We shall not consider the possibility that α may differ between the two species, since this would represent a large relative difference between the corresponding particles' respective Lagrangians.

To study the oscillations, we must determine the energy of a kaon in the frame of quantization, in which a_H^μ is purely spacelike. We then boost into the rest frame of the particle. In the quantization frame, the energy for a scalar particle with mass m , momentum \vec{k} , and a Lorentz-violating parameter a_H^μ is approximately

$$E \approx \sqrt{m^2 + \vec{k}^2} + \frac{\alpha |\vec{a}_H| |k_{\hat{a}_H}|}{\sqrt{m^2 + \vec{k}^2}}. \quad (11)$$

This is valid to first order in $|\vec{a}_H|$, and henceforth, we shall neglect any higher-order corrections, which should be miniscule. The group velocity \vec{v}_g corresponding to the energy E is then

$$\vec{v}_g = \frac{\vec{k}}{\sqrt{m^2 + \vec{k}^2}} + \frac{\alpha |\vec{a}_H|}{\sqrt{m^2 + \vec{k}^2}} \left[\text{sgn}(k_{\hat{a}_H}) \hat{a}_H - \frac{|k_{\hat{a}_H}| \vec{k}}{m^2 + \vec{k}^2} \right]. \quad (12)$$

This corresponds to a Lorentz factor of

$$\gamma = \frac{1}{\sqrt{1 - \vec{v}_g^2}} = \frac{\sqrt{m^2 + \vec{k}^2}}{m} \left(1 + \frac{\alpha |\vec{a}_H| |k_{\hat{a}_H}|}{m^2 + \vec{k}^2} \right). \quad (13)$$

So the rest energy, $E_0 = \gamma(E - \vec{v}_g \cdot \vec{k})$, is simply

$$E_0 = m + \frac{\alpha |\vec{a}_H| |k_{\hat{a}_H}|}{m}. \quad (14)$$

The rate of oscillations is determined by the difference between the rest energies of the K_1 and K_2 modes. If $\frac{|\Delta \vec{a}_H|}{|\vec{a}_H|}$ is small, then the signs of $k_{\hat{a}_{H1}}$ and $k_{\hat{a}_{H2}}$ will almost always be the same; we shall for now neglect any slight deviations from this around $k_{\hat{a}_H} \approx 0$. With this approximation, the energy difference is

$$\Delta E = m_1 - m_2 + \alpha \text{sgn}(k_{\hat{a}_H}) \vec{k} \cdot \left(\frac{m_2 \vec{a}_{H1} - m_1 \vec{a}_{H2}}{m_1 m_2} \right). \quad (15)$$

The most obvious signature for this form of Lorentz violation would be sidereal variations in the oscillation rate. We shall consider this variation in two different regimes. The momentum \vec{k} consists of two parts—the momentum \vec{k}_\oplus due to the motion of the earth relative to the quantization frame and the momentum \vec{k}_L of the kaons measured in the laboratory frame. If $\vec{k}_\oplus \cdot \hat{a}_H \gg |\vec{k}_L|$, then the motion of the earth is the dominant effect. If the quantization frame is the rest frame of the cosmic microwave background, then the velocity scale corresponding to \vec{k}_\oplus is $365 \text{ km} \cdot \text{s}^{-1}$. In this regime, $\text{sgn}(k_{\hat{a}_H}) = \text{sgn}(k_\oplus \cdot \hat{a}_H)$ is a constant. If the laboratory momentum \vec{k}_L has component $k_{L,z}$ along the polar (z -) axis and a component of magnitude $k_{L,xy}$ in the equatorial (xy -) plane, and $a_{Hj,z}$ and $a_{Hj,xy}$ are the corresponding components of \vec{a}_{Hj} , then the time variation of ΔE is given by

$$\begin{aligned} \Delta E = & \left[m_1 - m_2 + \alpha \text{sgn}(k_\oplus \cdot \hat{a}_H) \left(\vec{k}_\oplus \cdot \frac{m_2 \vec{a}_{H1} - m_1 \vec{a}_{H2}}{m_1 m_2} + k_{L,z} \frac{m_2 a_{H1,z} - m_1 a_{H2,z}}{m_1 m_2} \right) \right] \\ & + \alpha \left[\text{sgn}(k_\oplus \cdot \hat{a}_H) k_{L,xy} \frac{m_2 a_{H1,xy} - m_1 a_{H2,xy}}{m_1 m_2} \right] \cos(\omega_\oplus t + \psi). \end{aligned} \quad (16)$$

Here, ω_\oplus is the earth's sidereal rotation frequency, and ψ is a phase determined by the initial conditions. In this case, the time-dependence of the oscillation frequency is entirely sinusoidal, and the effects are essentially indistinguishable from those generated by a local Lorentz-violating term, because the absolute value in the dispersion relation has no direct effect.

If, on the other hand, the magnitude of the laboratory momentum \vec{k}_L is much larger than $\vec{k}_\oplus \cdot \hat{a}_H$, then the situation is more complicated, because the sign of $k_{\hat{a}_H}$ may change. Let θ_{a_H} and θ_{k_L} be the colatitudes corresponding to the directions of \vec{k}_L and \vec{a}_H . If we neglect any contributions from \vec{k}_\oplus , then the sign of $k_{\hat{a}_H}$ will change during the course of the earth's rotation exactly if $\cos^2 \theta_{a_H} < \sin^2 \theta_{k_L}$ (i.e., if $0 \leq \theta_{a_H} \leq \frac{\pi}{2}$, then there will be a change in $\text{sgn}(k_{\hat{a}_H})$ if $|\frac{\pi}{2} - \theta_{k_L}| < \theta_{a_H}$). The energy difference is thus

$$\begin{aligned} \Delta E = & m_1 - m_2 + \alpha \left| \frac{k_{L,z} a_{H1,z}}{m_1} + \frac{k_{L,xy} a_{H1,xy}}{m_1} \cos(\omega_\oplus t + \psi_1) \right| \\ & - \alpha \left| \frac{k_{L,z} a_{H2,z}}{m_2} + \frac{k_{L,xy} a_{H2,xy}}{m_2} \cos(\omega_\oplus t + \psi_2) \right|. \end{aligned} \quad (17)$$

The two phases ψ_1 and ψ_2 should be nearly equal; differences between them are suppressed by further factors of $\frac{|\Delta \vec{a}_H|}{|\vec{a}_H|} \ll 1$. If $\cos^2 \theta_{a_H} \geq \sin^2 \theta_{k_L}$, the time variation of ΔE is simply sinusoidal, as it was in the case in which \vec{k}_\oplus dominated. However, if $\cos^2 \theta_{a_H} < \sin^2 \theta_{k_L}$, then ΔE has cusps at $\cos(\omega_\oplus t + \psi_j) = -\frac{k_{L,z} a_{Hj,z}}{k_{L,xy} a_{Hj,xy}}$. These cusps represent the most telling signature indicating the presence of an a_H^μ interaction in the neutral kaon system. In fact, if $k_{L,z} a_{H1,z} \approx k_{L,z} a_{H2,z}$ is small enough to be neglected, then the oscillations in ΔE will effectively have period $\frac{\pi}{\omega_\oplus}$, rather than $\frac{2\pi}{\omega_\oplus}$, because the time-dependent part of ΔE will

be proportional to $|\cos(\omega_{\oplus}t + \psi)|$. This kind of effect could not be generated by a simple a^μ coefficient.

Because the Hilbert transform interaction we have considered is even under C, P, and T, it does not contribute directly to the quantities parameterizing the CP and CPT violations in the meson system. It is most natural, therefore, to search for the effects of a_H^μ through a direct analysis of the K^0 - \bar{K}^0 oscillation rate. (It is for this reason that we have neglected any decay and CP-violating processes in our analysis, as they do not affect this kind of analysis in any fundamental way.) A beam that initially consists entirely of K^0 will contain, after a time t , fractions $\cos^2\left(\frac{\Delta Et}{2}\right)$ and $\sin^2\left(\frac{\Delta Et}{2}\right)$ of K^0 and \bar{K}^0 particles, respectively. By measuring the behavior of the beam as a function of t , one may determine the time dependence of its particle content. If the oscillation frequency $\frac{\Delta E}{2}$ shows time and directional dependences of the type displayed in (17), then this will be strong evidence for this type of nonlocal Lorentz violation.

The current best measurements of the K_S - K_L mass difference give results of approximately $3.48 \pm 0.01 \times 10^{-6}$ eV [34], compared with a neutral kaon mass of 498 MeV. For KTeV kaons, with total energies of 70 GeV, the implied sensitivity for $\frac{\Delta m}{m}|\vec{a}_H|$ or $|\Delta\vec{a}_H|$ is of the order of 10^{-10} eV. If $\frac{\Delta m}{m}|\vec{a}_H|$ generates the dominant contribution, then the direct sensitivity for $|\vec{a}_H|$ is only at the 10^4 eV level. These potential experimental constraints are thus much less stringent than those that can be obtained for the CPT-violating a^μ .

In this paper, we have presented a new possible Lorentz-violating interaction, beyond that previously considered in the SME. This interaction, which involves a Hilbert transform, is superficially renormalizable, and, although it is weakly nonlocal, it could reasonably be generated by some more complicated underlying theory. The corresponding nonlocal theories have causality and stability properties similar to those of local Lorentz-violating theories. However, the Hilbert transform gives rise to a form of Lorentz violation that is CPT even, yet which possesses a vector index; this is a property that has not previously been observed in the study of Lorentz violation. We have also considered the effects such an interaction might have on neutral meson oscillations and shown that, when the laboratory momentum is large and aligned appropriately, this interaction could modify the usual oscillations in a unique way. Finally, since the interactions we have considered are not even the most general that could be constructed with a Hilbert transform, there may be many more weakly nonlocal Lorentz-violating interactions with further interesting properties.

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